Field Goal for the Win

Objectives

- Students will explore the science and math behind kicking a field goal.
- Students will solve linear and quadratic equations to determine if the kick is made including applying the quadratic formula in a context.
- Students will use parametric equations graphically. (Note: no prior exposure to parametric equations is necessary. Students will explore each part separately before putting it all together.)
- Students will use trigonometric ratios to determine horizontal and vertical velocity components. (Note: students do not need any prior exposure to the trigonometric ratios as they are only part of the given equations for horizontal and vertical components of velocity.)

Vocabulary

- Initial Velocity
- Horizontal
- Vertical

About the Lesson

- In this activity, students will explore a parametric model of the flight of a ball graphically, numerically and algebraically. As a result, students will:

  Solve the $x(t)$ and $y(t)$ equations separately for values of t, then use that value of t to determine a coordinate pair.
- Students should know how to build a table of values given a function and inputs.
- Students should recognize that cosine and sine of an angle yields decimal values.
- Teachers may want to give more guidance regarding parametric equations or trigonometric values using the pre-assignment handouts provided.

Activity Materials

- Compatible TI Technologies:

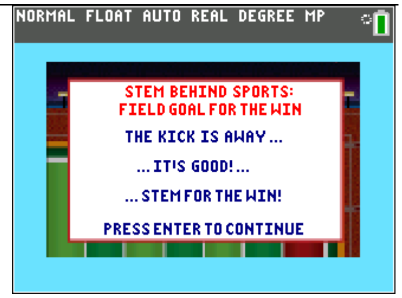
  TI-84 Plus*

  TI-84 Plus Silver Edition*

  TI-84 Plus C Silver Edition

  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

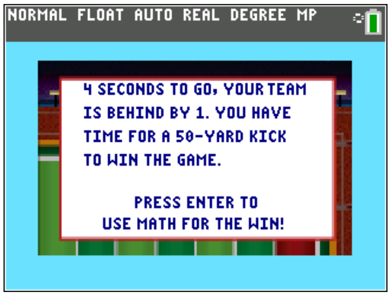
- GE_The_Kick_Student.pdf
- GE_The_Kick_Student.doc
- The_Kick_color.8xp
- Image0.8ca
- Image9.8ca

TI-84 Plus*, and TI-84 Plus Silver Edition* use

- The_Kick.8xp

Tech tip: Make sure when sending the program file The_Kick_color.8xp to your TI-84 Plus CE calculators that the program, and image0 and image9 are sent. For TI-84 Plus or TI-84 Plus Silver Edition users, send the program The_Kick.8xp without the image files.

Introduction



In this activity, students will use a mathematical model of a field goal kick. Students first explore the model graphically on the calculator to understand how the horizontal and vertical positions are related to time. Students then break the kick into its horizontal and vertical components and use algebra to explore those separately. Finally, students put it all together and determine if their kick is made and wins the game. There are extensions for exploring student created kicks as well as a scenario that involves modeling the defense.

Teaching Tips and Background:

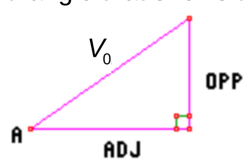
- Use TI-Connect CE to deliver the program and background images to your teacher calculator. Download the latest version at: https://education.ti.com/en/us/software/details/en/CA9C74CAD02440A69FDC7189D7E1B6C2/swticonnectcesoftware
- For extra information about sending programs and images using TI-connect CE download the guidebook at the link below: https://education.ti.com/en/us/guidebook/details/en/9A4FE63E3B054CB49C06B202578AB7FE/ti-connect-ce

- Guide your students on opening and running the program THEKICK. While the program is running the circle in the upper right corner of the screen will show whether the program is waiting (white) or computing (yellow). If there is no circle, the program is not running.
- While in the program students will navigate using the arrow keys, number keys, and the enter key, to control the program. To exit the program and explore, choose Option 7 from the submenus. At the end of the activity, make sure students run the program and select Quit. This will set the calculator to the default settings.
- Students should have an opportunity (between 2 and 5 minutes) to explore different values for kicks before jumping into question 1. Have them run Option three from the main menu for several different kick lengths, angles, and velocities. Note that kick lengths must be between 20 and 70 yards, angles should be between 0 and 90 degrees, and kick velocities are between 50 and 90 ft/s.
- The mathematical background section is information for you as a teacher. Share with your students as much or as little as you feel comfortable.

Mathematical background for the model:

In order to model the motion of the ball during its flight, the model makes use of parametric equations and trigonometric functions. Students do not need prior experience with trigonometry or parametric equations in order to explore this lesson graphically or algebraically. A pre-work exploration is provided in order to familiarize students with the values of the trigonometric functions cosine and sine, for angles between 0 and 90 degrees. This will allow students to be comfortable with the fact that $\cos(43°)$ is just a decimal value between 0 and 1. The pre-work can also be used to show how two values can depend on one independent variable at the same time. This will allow them to see how the horizontal and vertical positions can be shown as functions of time.

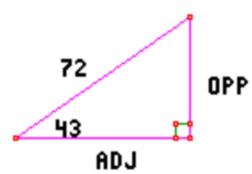
The horizontal and vertical positions of the ball after it is kicked depend on many factors (wind, air resistance, elevation, gravity, etc.). For simplicity and accessibility, in this model we are only including the effect of gravity on the ball. The kick can be broken into its horizontal and vertical components using a triangle that shows the initial velocity as the hypotenuse.



Using the trigonometric ratios:

$\cos(A) = \dfrac{adj}{V_0}$ $\qquad$ $\sin(A) = \dfrac{opp}{V_0}$

$adj = V_0 \cdot \cos(A)$ $\qquad$ $opp = V_0 \cdot \sin(A)$



Horizontal velocity component:

$\cos(43°) = \dfrac{adj}{72}$

$adj = 72 \cdot \cos(43°)$

Vertical velocity component:

$\sin(43°) = \dfrac{opp}{72}$

$opp = 72 \cdot \sin(43°)$

Since $\text{distance} = \text{rate} \cdot \text{time}$, the horizontal distance traveled (position), *x*(*t*), is given by : $x(t) = V_0 \cdot \cos(A) \cdot t$ where V_0 is the initial velocity, *A* is the angle of the kick, and *t*, is the time in seconds since the ball was kicked. In this example $x(t) = 72 \cdot \cos(43°) \cdot t$

For the vertical motion, the effect of gravity must be considered. In order to get the vertical position as a function of time, we have to add the effect of gravity to the vertical velocity of the kick. From physics it is known that the effect of gravity on position is $-\frac{1}{2} g \cdot t^2$, where $g = 32 \frac{\text{ft}}{\text{s}^2}$. The vertical positon of the ball at a time, *t*, can be found using the function $y(t) = V_0 \sin(A) \cdot t - 16 \cdot t^2$. For the above example: $y(t) = 72\sin(43°) \cdot t - 16 \cdot t^2$

Some information about football that students may need for this activity:

- Remind students there are 3 feet in a yard.
- The crossbar of the goalposts is 10 feet high, and is directly above the back line of the end zone.
- The end zone is 10 yards deep.
- Line of scrimmage is the yard line where the ball is placed before the play begins.
- Professional kickers kick the ball from 7 yards behind the line of scrimmage.
  From this information, when the line of scrimmage is the 33 yard line, a field goal attempt would be a 50 yard kick. (33+10+7=50)

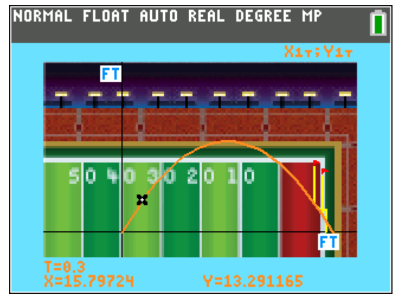
Get Your Game On

Four seconds left in the Big Game and the score is 27 to 28 - your team is behind. A 50-yard field goal wins the game. Your task is to create a mathematical model to demonstrate kicking a field goal to win the game.

Press [prgm] and run the program THEKICK.

1. <u>The Kick</u>

Follow the on-screen prompts to the main menu.
Press 1 to see the 50-yard kick. Then press [trace].



a. Investigate the information on the screen. Notice the three variables, T, X, and Y. Press the right arrow three times and record the values of the variables to the nearest thousandth.

<u>Answer:</u> *T*= 0.3 ; X= 15.797 ; *Y*= 13.291

b. Interpret the values of these variables in context, including units.

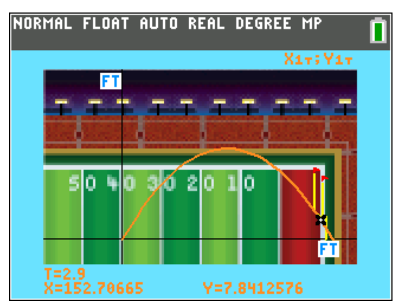
<u>Answer:</u> *T* is time in seconds after the ball is kicked; *X* is horizontal position downfield from where the ball is kicked; *Y* is height above the ground.

- Carefully monitor student discussion and guide them to the correct interpretations of *T*, *X*, and *Y* on the graphical model.

c. Trace on the function to a different point and stop. Record T, X, and Y, then discuss what these values mean with your group.

<u>Answer:</u> Answers will vary.

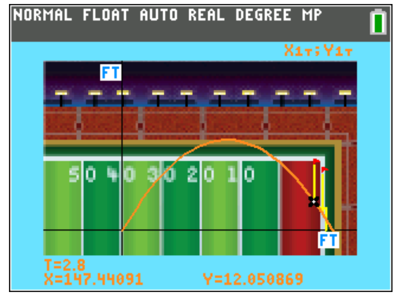
d. Using the trace feature, graphically investigate the time when the ball is 150 feet from where it was kicked in the horizontal direction. Between which two values of T does the ball pass 150 feet downfield? Record your answers to the nearest tenth of a second. How high is the ball at each of these times?



<u>Answer:</u> Between *T*=2.8 and 2.9 seconds. The height of the ball is 12.051 ft. at 2.8 seconds and 7.841 ft. at 2.9 seconds.

1. The Kick (continued)

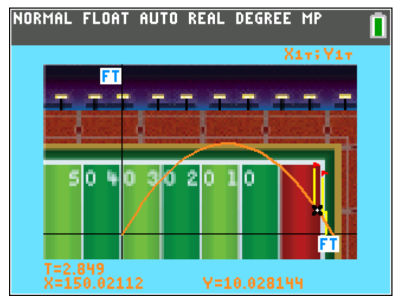
e. Using the trace feature, graphically investigate a time when the ball is 10 feet high. Between which two values of T does the ball reach 10 feet high? Record your answers to the nearest tenth of a second. How far downfield is the ball at each of these times?



Answer: *T*= 0.2 and 0.3 seconds and again at 2.8 and 2.9 seconds. The ball is 10.531 ft downfield at 0.2 seconds and 15.797 ft downfield at 0.3 seconds. The ball is 147.440 ft downfield at 2.8 seconds and 152.707 ft downfield at 2.9 seconds.

- Note the second time it reaches 10 feet will be the more important investigation.

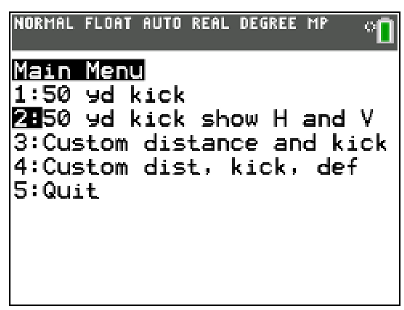
f. **Answer:** It appears that the kick may have been good but it is very difficult to tell graphically. Yes, we can prove it graphically by zooming in to *T*=2.849 seconds. The ball is 150.021 ft downfield and 10.028 ft above the ground, which is still slightly higher than the 10-foot crossbar on the goal posts. This means that the kick just barely cleared the crossbar.



- Students should note that they can input values for *T* and “zoom in” on a specific point in time to observe the desired values for *X* or *Y* by increasing the precision of their guesses.

2. Modeling Horizontal Motion

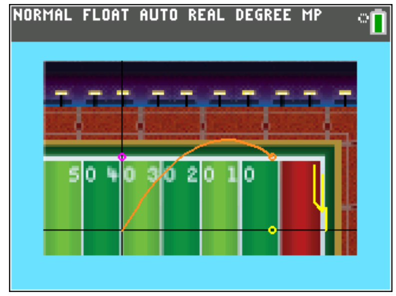
In order to answer question 1f. algebraically instead of on the graph, we need a function to model the horizontal distance the ball travels downfield after it’s kicked as a function of time, $x(t)$.



Run the program THEKICK.

On the main menu press 2 to see a kick with the horizontal and vertical components shown. The angle of the kick is 43 degrees and the velocity of the kick is 72 ft/s. Press enter and watch the kick. You will be placed into Trace mode. Press enter when you are done exploring.

2. Modeling Horizontal Motion (continued)



When students run the program and choose Option two, make sure they pay attention to the motion of the horizontal component. Particularly its speed. Also note that after the graph shows they are in a modified trace mode. They won't be able to enter values, but they can trace on the path. Pressing [enter] returns them to a submenu from which they can choose to exit and explore using trace, they can repeat the kick, or return to the main menu.

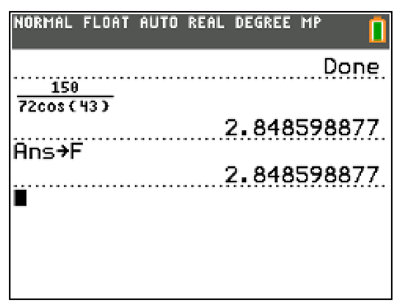
- Note for 84 Plus users there is no variation of color.

a. Repeat the kick and watch the horizontal component (yellow). Does its speed increase, decrease, or remain constant? What does that tell you about the kind of equation that will model the horizontal motion?

Answer: The speed remains constant which indicates that the function/equation that will model this motion is linear in nature. Solution will be of the form $x(t) = V_h \cdot t$ where V_h is the horizontal velocity.

b. The function, for the horizontal distance traveled downfield, if the ball is kicked with initial velocity 72 ft/s and an angle of 43 degrees is $x(t) = 72 \cdot \cos(43°) \cdot t$. Is $x(t)$ linear or nonlinear?

Answer: $x(t)$ is linear since $72\cos(43°)$ is a constant.



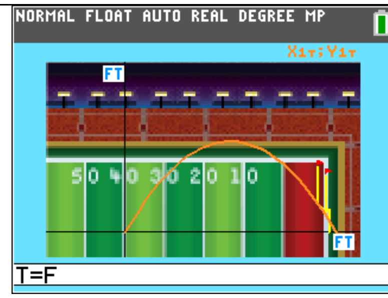
c. How long will it take for the ball to travel 150 feet downfield? Exit the program, then solve $x(t) = 150$ and store your answer in variable F.

Answer: $150 = 72 \cdot \cos(43°) \cdot t \rightarrow t = \dfrac{150}{72 \cdot \cos(43°)}$

So $t \approx 2.8485989$ seconds

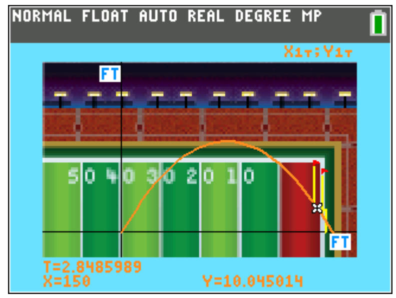
Tech tip: Storing a value using the [sto→] key will allow students to trace directly to their solution.

d. Press [trace]. Using the trace feature, trace to the value for F and explain what the numbers on the screen mean.



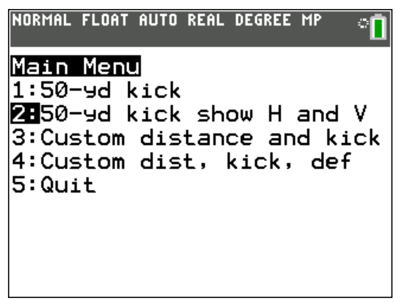
Answer: *T*=2.8486 seconds after the ball is kicked, the ball passes over the crossbar just barely. *Y*=10.045 ft.

e. Did you make the kick and win the game? How do you know?



Answer: Since the height of the ball is slightly greater than 10 feet, the ball just passes over the crossbar at 150 feet (50 yards) downfield. The kick is good and we win.

3. Modeling Vertical Motion



A second way to answer question 1f. algebraically instead of graphically, uses a function to model the height of the ball during a kick as a function of time, $y(t)$.

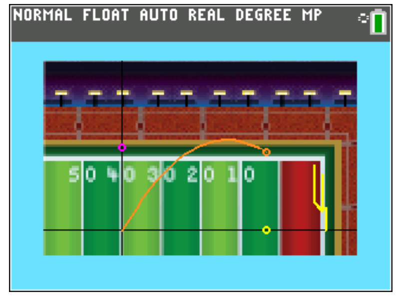
Run the program THEKICK.
On the main menu press 2 to see a kick with the horizontal and vertical components shown. The angle of the kick is 43 degrees and the velocity of the kick is 72 ft/s. Press [enter] and watch the kick. You will be placed into Trace mode. Press [enter] when you are done exploring.

When students run the program and choose Option 2, be sure this time, they are paying attention to the vertical component (light pink). Its speed and position will model only the vertical motion of the kick. Students should note that it starts off fast but slows to a stop and then reverses motion speeding up again as it falls to the ground.

- Note for 84 Plus users the colors are all the same.

3. Modeling Vertical Motion (continued)

a. Repeat the kick (Option 1 in the submenu) and this time, pay attention to the vertical component (light pink). Describe its motion. Include its speed and direction in your description. Does its speed increase, decrease, remain constant? What does that tell you about the kind of equation that will model the vertical component?



Answer: Its speed first is decreasing, even goes to zero, and then begins increasing as it falls back to the ground. The equation should be nonlinear.

b. What other factors besides the kick contribute to the vertical position of the ball after it's kicked?

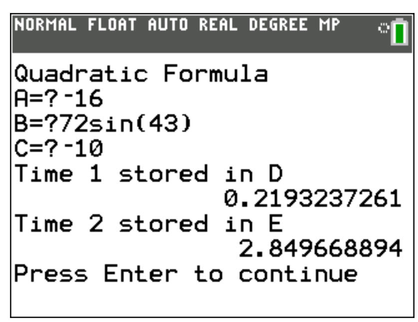
Answer: Gravity will affect the vertical motion.

Acceleration due to gravity is -32 ft/s^2. It is negative because the ball is being pulled down toward the ground. When modeling vertical position, we can apply physics laws to know we can add the initial velocity multiplied by the time (this gives how high the ball would be if there were no gravity) and $-\frac{1}{2}g \cdot t^2$. The function for modeling the ball's height, $y(t)$, if it is kicked with an initial velocity of 72 ft/s at an angle of 43 degrees is: $y(t) = 72 \cdot \sin(43°) \cdot t - 16 \cdot t^2$.

c. What kind equation is $y(t)$? Is it linear or nonlinear?

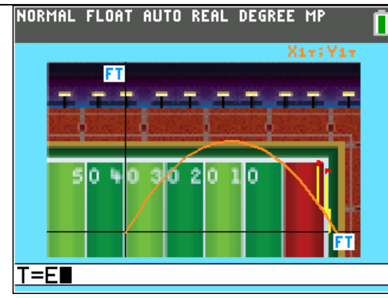
Answer: The equation is quadratic (students may say parabolic). This means it is nonlinear as we predicted.

d. Solve for $y(t) = 10$ to determine when the ball is 10 feet in the air. (Hint: You may want to use the quadratic formula Option in the program. It is Option 5 in the submenu after you've drawn the graph.) Use the trace feature to confirm your solution.

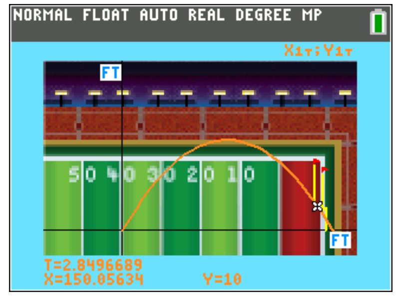


Answer: There are two times when the ball is 10 feet above the ground. First at approximately 0.2193 seconds and again at approximately 2.8497 seconds. The second is the one we are concerned about.

e. Explain how your solution is consistent to your answer from 2e.

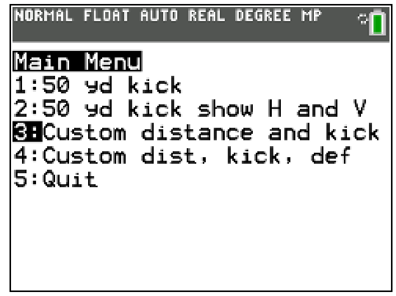


Answer: At 2.8496688 seconds the ball is 10 feet above the ground and has passed the goal posts since it is 150.056 ft downfield. Since the ball is falling, it must have been higher than 10 feet as it passed the crossbar. This confirms the result from problem 2 part e.



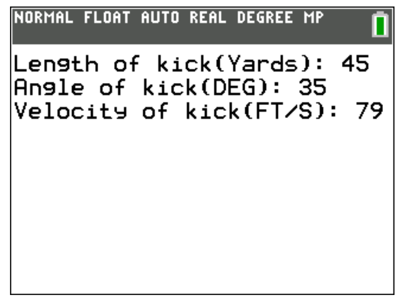
Extensions

4. Application of the model



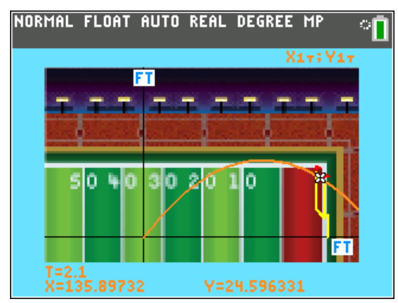
Run the program THEKICK again and select Option 3. You're going to kick a field goal to win the game. Professional kickers kick the ball with a velocity of about 70 to 88 ft/s (48 to 60 mph) and at an angle that varies between 27 and 43 degrees. Choose your velocity and kick angle and run the program to graph your kick.

Length of kick (yards): __45__ Angle (degrees): ___35______
Velocity (ft/s): _____79______



The equations you used in questions 2 and 3 to model the horizontal and vertical positions of the ball can be used at the same time to model the actual flight of the ball after it is kicked.

Student answers will depend on their scenario. Solutions here are based on a 45-yard field goal, an angle of 35 degrees, and a kick velocity of 79 ft/s.



a. Based on the graph, can you tell if the ball passes above the 10-foot crossbar on the goal posts? How can you tell?

Answer: Since the path of the ball is much higher than the goal post, it appears that the kick was made. Also using trace, at 2.1 seconds the ball is 135.897 ft (roughly 45 yards) downfield and 24.596 ft in the air, well above the 10 ft crossbar.

b. Fill in the equation for $x(t)$ and solve algebraically for when the ball passes through the goal posts. Use your solution to decide if the field goal is made or not.

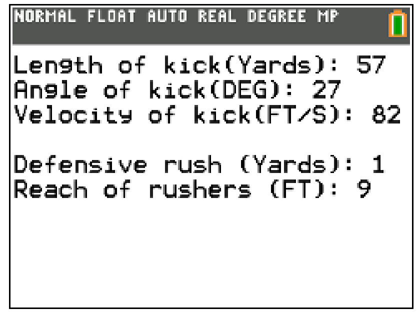
Answer: $x(t) = \_79\_\cos(\_35\_)\cdot t$. Using this equation: $135 = 79\cdot\cos(35)\cdot t$ and solve for *t.* $t = \dfrac{135}{79\cos(35)}$ which yields $t = 2.0861$ seconds. Using the trace key or the equation for $y(t)$ we can show that the height of the ball after 2.0861 seconds is approximately 24.897 ft.

c. Attempt the kick with a different angle or velocity and determine graphically if the kick is made using the program. Then use algebra to confirm your answer.

Answer: Answers will vary based on student inputs.

5. <u>But what about the defense?</u>

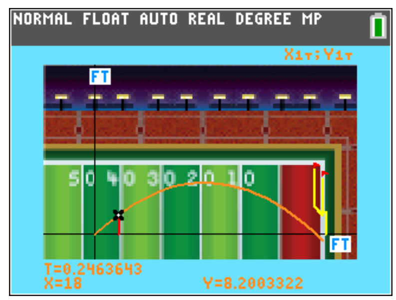
Run the program THEKICK again and select Option 4. You're going to kick a field goal to win the game. Professional kickers kick the ball with a velocity of about 70 to 88 ft/s (48 to 60 mph) and at an angle that varies between 27 and 43 degrees. Choose your velocity and kick angle and run the program to graph your kick. The kicker kicks from 7 yards behind the line of scrimmage and the defense typically gets little or no rush (between 0 and 2 yards), and defensive playerscan reach about 8 to 9 feet in the air.



Length: 57 Angle: 27 Velocity: 82 Rush: 1 Reach: 9

Student answers will depend on their scenario. Solutions here are based on a 57-yard field goal, an angle of 27 degrees, a velocity of 82 ft/s, a rush of 1 yard, and a reach of 9 ft.

a. Based on your model, will a defender block the kick? Defend your answer graphically and algebraically.



<u>Answer:</u> Based on the graph, yes the defender blocks the kick since the path of the ball passes through the red line which represents the defense.

$$18 = 82\cos(27)t$$

$$t = \frac{18}{82\cos(27)}$$

$$t = 0.2464$$

Another way to answer is to see what the height of the ball would be at 6 yards downfield (7 yards minus 1 yard rush). This yields a height of 8.2 ft which is less than the 9 ft reach. The kick is blocked.

b. Based on your model, will the ball pass above the 10-foot crossbar on the goal posts? How can you tell? Defend your answer more than one way.

<u>Answer:</u> The kick would have been short based on the graph. Solving $171 = 82\cos(27)t$ for t yields a height of approximately zero so the ball falls short.

c. Attempt the kick with a different angle or velocity and determine algebraically if the kick makes it over the defense and is good. Then use the program to confirm your answer.

<u>Answer:</u> Answers will vary based on student inputs.